

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2

Branch: B.Tech (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions:

(14)

- a) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x \, dx$ is
(A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) zero (D) $\frac{16}{35}$
- b) If $f_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then $(f_n + f_{n-2})$ is equal to _____.
(A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$
- c) $B(1,1) =$ _____
(A) 1 (B) 0 (C) 1/2 (D) none of these
- d) $\frac{1}{4} \sqrt{\frac{3}{4}} =$ _____
(A) $\frac{\pi}{\sqrt{2}}$ (B) $\pi\sqrt{2}$ (C) $\sqrt{2\pi}$ (D) none of these
- e) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-2\sin^2\theta}}$ is equal to
(A) $\frac{1}{\sqrt{2}} E\left(\frac{1}{\sqrt{2}}\right)$ (B) $\frac{1}{2} K\left(\frac{1}{\sqrt{2}}\right)$ (C) $\frac{1}{\sqrt{2}} K\left(\frac{1}{\sqrt{2}}\right)$ (D) $\frac{1}{2} E\left(\frac{1}{\sqrt{2}}\right)$
- f) $\operatorname{erf}(-x)$ is
(A) an odd function (B) an even function (C) neither odd nor even function
(D) none of these
- g) If the power of x are even, then the curve is symmetrical about
(A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these



- h) The curve $x^3 + y^3 = 3axy$ represent
 (A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid
 (D) Folium of Descartes
- i) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx dy$ is equal to
 (A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these
- j) On converting into polar coordinates $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dx dy$ is equal to
 (A) $\int_0^{\pi/2} \int_0^{2a\cos\theta} r dr d\theta$ (B) $\int_0^{\pi/2} \int_0^{2a\sin\theta} r dr d\theta$ (C) $\int_0^{\pi/2} \int_0^{2a\sin\theta} r dr d\theta$ (D) none of these
- k) The degree and order of the differential equation of all parabolas whose axis is x-axis are
 (A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these
- l) The solution of the equation $(x+y)^2 \frac{dy}{dx} = a^2$ is
 (A) $(x+y) - a \tan^{-1}\left(\frac{x+y}{a}\right) = x+c$ (B) $(x+y)a \tan x = x+c$
 (C) $(x+y) - a \cos a = x+c$ (D) none of these
- m) The infinite series $1+r+r^2+\dots+r^{n-1}+\dots$ is divergent if
 (A) $|r| < 1$ (B) $|r| > 1$ (C) $r \geq 1$ (D) $r = -1$
- n) The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
 (A) $\log 2$ (B) zero (C) infinite (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Using reduction formula prove that $\int_0^{\pi} x \sin^7 x \cos^4 x dx = \frac{16\pi}{1155}$ (5)

b) Evaluate: $\int_0^{\infty} x^4 e^{-x^4} dx$ (5)

c) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ (4)

Q-3 Attempt all questions (14)

a) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \cdot \frac{\left| \frac{1}{n} \right|}{\left| \frac{1}{n} + \frac{1}{2} \right|}$ (5)

b) Solve: $xdy - ydx = \sqrt{x^2 + y^2} dx$ (5)



c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$. (4)

Q-4 Attempt all questions (14)

a) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - y^2}} (x^2 + y^2) dx dy$ by changing into polar coordinates. (5)

b) Test for convergence the series $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$. (5)

c) Evaluate: $\int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^2 6\theta d\theta$ (4)

Q-5 Attempt all questions (14)

a) Solve: $\frac{dy}{dx} + 2y \tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$. (5)

b) Change the order of integration in the integral $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$ and evaluate it. (5)

c) Prove that $B(m, n) = B(m, n+1) + B(m+1, n)$ (4)

Q-6 Attempt all questions (14)

a) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$ (5)

b) Using reduction formula prove that $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2} \left(\log 2 - \frac{1}{2} \right)$. (5)

c) Solve: $(y^2 e^{xy^2} + 4x^3) dx + (2xye^{xy^2} - 3y^2) dy = 0$ (4)

Q-7 Attempt all questions (14)

a) Trace the curve $y^2(2a - x) = x^3$. (5)

b) Find the volume of the solid generated by the revolution of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$ about the x - axis. (5)

c) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}}$ (4)

Q-8 Attempt all questions (14)

a) For small values of x , show that $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{1!3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + \dots \right)$. (5)

b) Trace the curve $r^2 = a^2 \cos 2\theta$. (5)

c) Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$. (4)

